Sydney
Girls
High
School

2019	TRIAL HIGHER SCHOOL CERTIFICATE **EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations. A correct answer without working will be awarded a maximum of 1 mark.

Total marks : 100

Section 1 – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II – 90 marks (pages **6 – 15**)

- Attempt Questions **11 – 16**
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name: …………………………………………………………... Teacher: …………………………………………………………	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2019 HSC Examination Paper in this subject.

Section 1

10 marks

Attempt questions 1-10

Use the Multiple-choice answer sheet for questions 1-10

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1. Which expression is equal to $\int \frac{dx}{\sqrt{-x(x+2)}}$?

   A. $\sin^{-1}(x+1)+c$

   B. $\sin^{-1}(1-x)+c$

   C. $\sin^{-1}(x-1)+c$

   D. $\sin^{-1}\left(\frac{x+1}{2}\right)+c$

2. What are the equations of the directrices of the hyperbola $9xy = 8$?

   A. $x+y=\pm\frac{16}{9}$

   B. $x+y=\pm\frac{4}{3}$

   C. $x-y=\pm\frac{16}{9}$

   D. $x-y=\pm\frac{4}{3}$

3. The cubic equation $x^3-3x^2+4x-1=0$ has roots α, β, γ .

   Which equation has roots $\frac{1}{\sqrt{\alpha}}, \frac{1}{\sqrt{\beta}}, \frac{1}{\sqrt{\gamma}}$?

   A. $x^6+4x^4-3x^2-1=0$

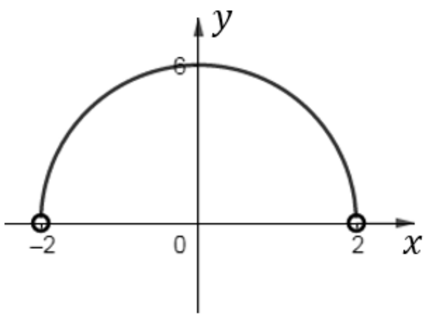
   B. $x^6-4x^4+3x^2+1=0$

   C. $x^6+4x^4-3x^2+1=0$

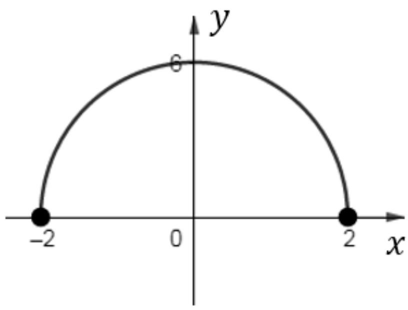
   D. $x^6-4x^4+3x^2-1=0$

4. Which graph best represents the curve $\frac{y}{\sqrt{36-9x^2}}=1$?

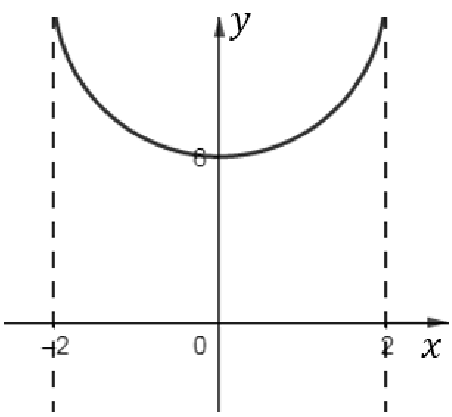
A.



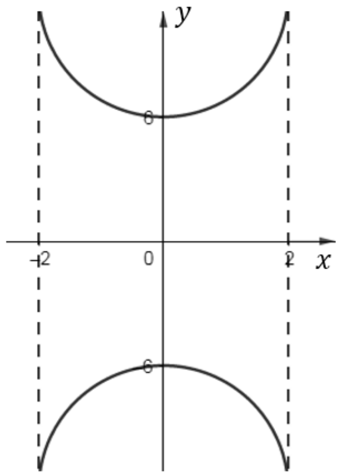
B.



C.



D.



5. Given that x is a negative integer , which of the following is equivalent to i^{4x-1}?

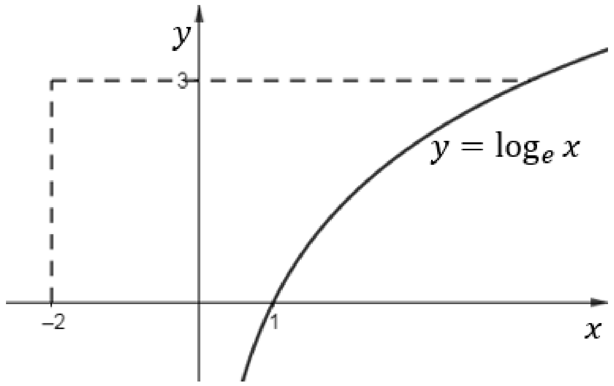
A. i

B. $-i$

C. 1

D. -1

6. The diagram below shows the graph $y = \log_e x$. The region bounded by $y = 3$, $y = \log_e x$, $y = 0$ and $x = -2$ is rotated about the line $x = -2$ to form a solid.



Which integral represents the volume of the solid formed?

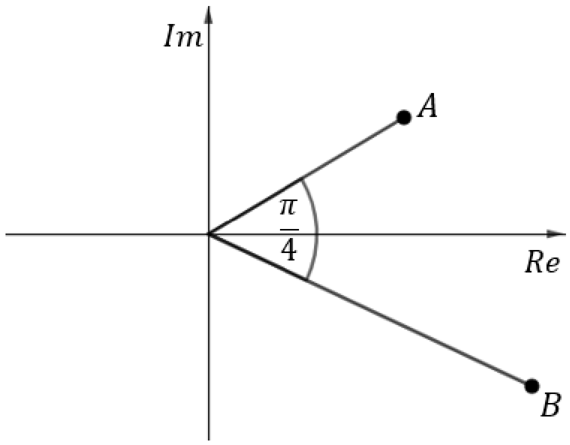
A. $\pi\int_0^3 (e^y + 2)^2 dy$

B. $\pi\int_0^3 (e^y - 2)^2 dy$

C. $\pi\int_{-2}^{\log_e 3} (\log_e x + 2)\, dx$

D. $\pi\int_{-2}^{\log_e 3} (\log_e x - 2)\, dx$

7.



In the diagram above, the point A represents the complex number Z.

The point B is represented by the complex number :

A. $(1 - i)Z$

B. $(1 + i)Z$

C. $1 + iZ$

D. $1 - iZ$

8. Consider the graph of $f(x) = x^{n+1} + x^{n+2} + x^{n+3} + \cdots + x^{2n}$ where n is an integer and $n \geq 2$. The graph has a horizontal point of inflexion if n is :

A. even

B. odd

C. a multiple of 3

D. a prime number

9. If $1 - i$ is a root of a polynomial $P(x)$ which has complex coefficients, then :

A. $P(1 + i) = 0$

B. $P'(1 - i) = 0$

C. $\frac{1}{P(1-i)} = 0$

D. $P(\sqrt{-2i}) = 0$

10. The general cubic equation $ax^3 + bx^2 + cx + d = 0$ where a, b, c and d are real numbers can only have one real solution if :

A. $b^2 > 3ac$

B. $b^2 < 3ac$

C. $b^2 \geq 3ac$

D. $b^2 \leq 3ac$

Section II

90 marks

Attempt questions 11-16

Start each question on a NEW piece of paper

Question 11 (15 marks)

Use a new sheet of paper.

(a) Let $z = 3 - 2i$ and $w = 2 + 3i$.

(i) Find $w - z$. 1

(ii) Express $\frac{|z|}{w}$ in the form $x + iy$ where x and y are real numbers. 2

(b) The equation $x^2(x + 4) = 3(x + 6)$ has only two unique solutions.

Find these solutions. 3

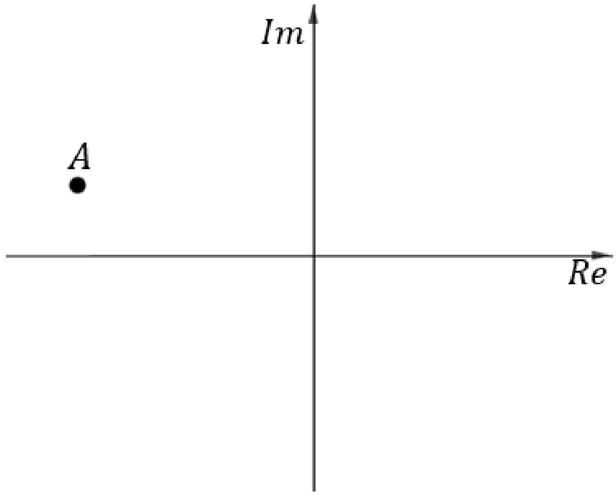
(c) By writing $\frac{x^3}{(x+1)(x-2)}$ in the form $ax + b + \frac{c}{x+1} + \frac{d}{x-2}$, find : 4

$$\int \frac{x^3}{(x+1)(x-2)}\, dx .$$

Question 11 continues on the next page

Question 11 (continued)

(d) On the Argand diagram below, the point A represents the complex number z such that $\arg(z) = \frac{11\pi}{12}$.



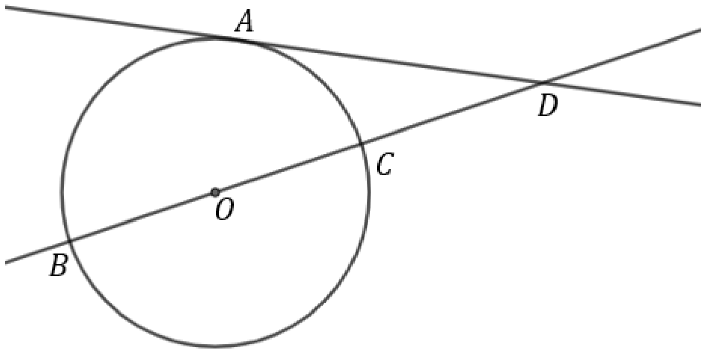
Copy the diagram and show on it the points which represent the complex number :

(i) $|z|$ 1

(ii) $-\frac{z}{i}$ 1

(iii) $(1-i)\,\bar{z}$ 1

(e) In the diagram below, the line AD is a tangent to the circle ABC with centre O.
The line BC passes through the points O and D .
Show that $AD^2 - CD^2 = BC \times CD$. 2



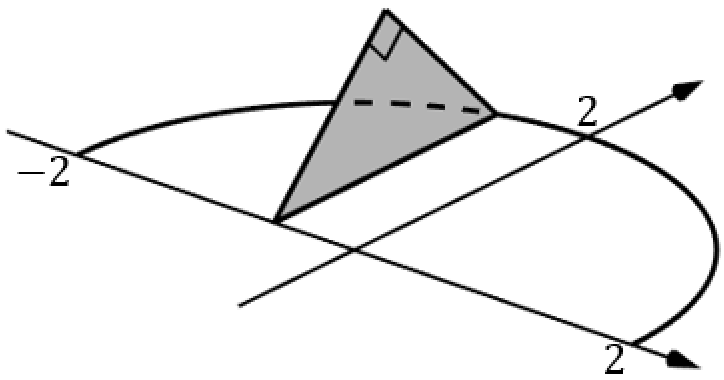
End of Question 11

Question 12 (15marks)

Use a new sheet of paper

(a) The base of a solid is the region enclosed by the semi-circle $y = \sqrt{4 - x^2}$ and the x axis. Each cross-section perpendicular to the x axis is a right angled isosceles triangle as shown in the diagram.

Find the volume of this solid. 3



(b) Find the gradient of the tangent to the curve $x^2 + xy^2 = 12$ at the point $(3, -1)$. 2

(c) Find : 3

$$\int \frac{x}{x^2 - 4x + 6}\, dx$$

Question 12 continues on the next page

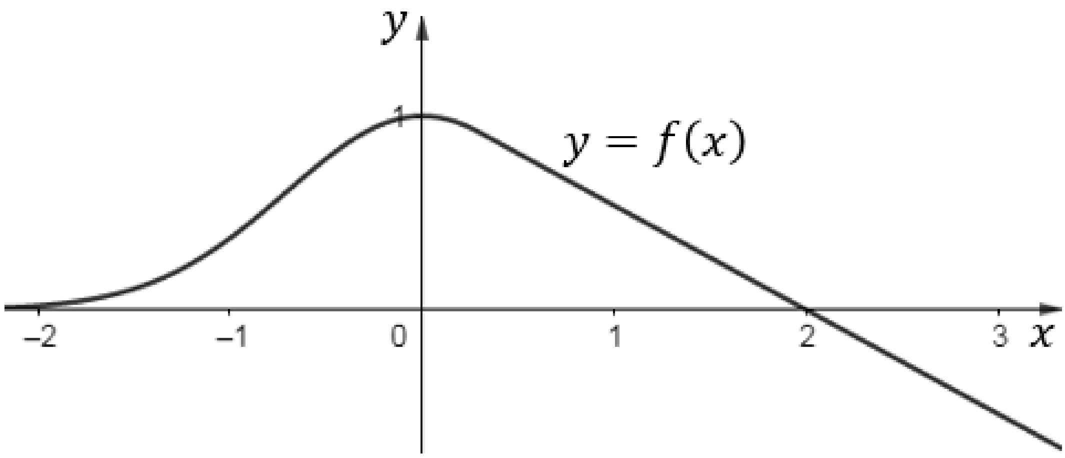
Question 12 (continued)

(d) The roots of $5x^3 + 7x^2 - 3x + 2 = 0$ are $\alpha,\ \beta$ and γ. 2

Find the value of :

$$(\alpha+\beta)^2 + (\alpha+\gamma)^2 + (\beta+\gamma)^2$$

(e) The diagram below shows the graph of the function $y = f(x)$. The x −axis is an asymptote for $x < 0$.



Draw a separate half page graph for each of the following functions, showing all asymptotes and intercepts.

(i) $y = f(|x|)$ 1

(ii) $y^2 = f(x)$ 2

(iii) $y = e^{f(-x)}$ 2

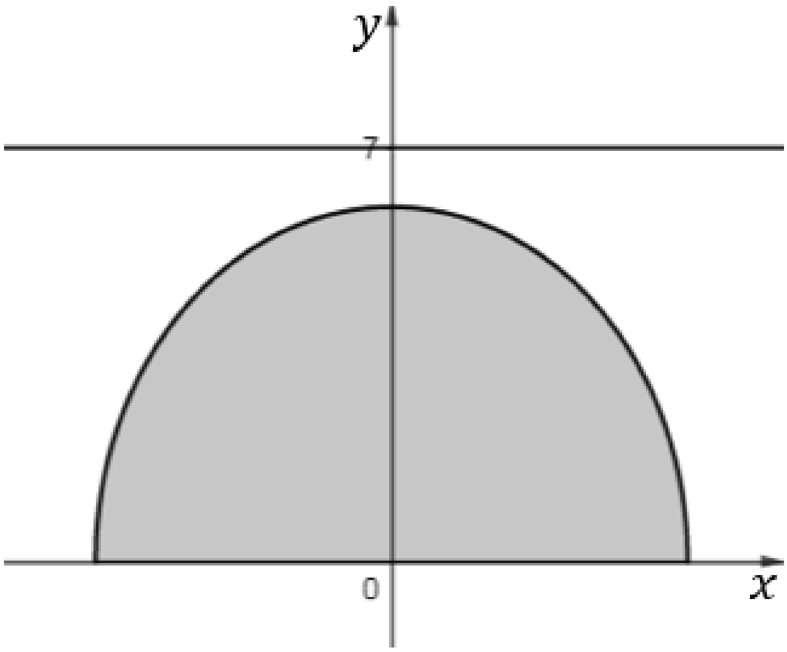
End of Question 12

Question 13 (15 marks)

Use a new sheet of paper

(a) The graph $y = \sqrt{36 - 4x^2}$ is shown in the diagram below. 3

Use the method of cylindrical shells to find the volume of the solid formed when the shaded region is rotated about the line $y = 7$.



(b) z is a complex number satisfying the equation $|z - \sqrt{6}| = 2\sqrt{2}$.

(i) Find the minimum value of $|z|$. 1

(ii) Find the maximum value of $\arg(z + \sqrt{6})$. 1

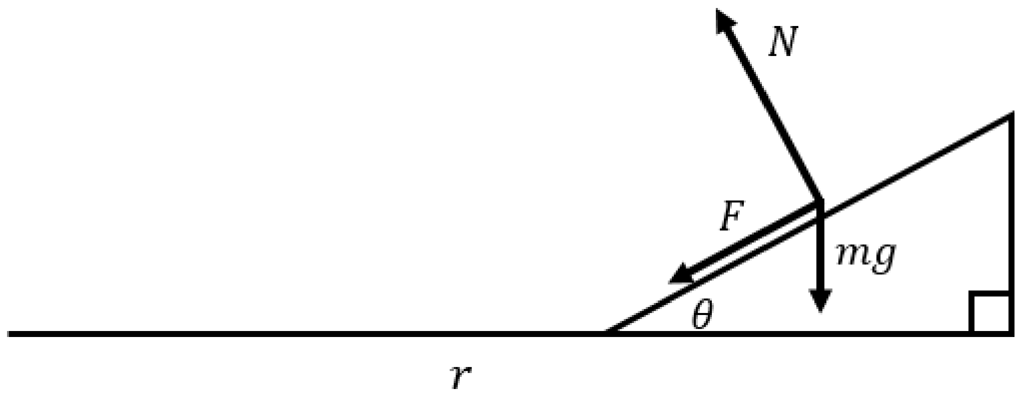
(iii) Simplify $|z - \sqrt{6} - 2\sqrt{2}|^2 + |z + 2\sqrt{2} - \sqrt{6}|^2$. 2

Question 13 continues on the next page

Question 13 (continued)

(c) A car of mass m is traveling at a speed v around a circular bend of radius r which is inclined at an angle θ to the horizontal. The forces on the car are the normal reaction N of the road, the friction F along the road and the gravitational force mg. By resolving the forces on the car, show that if the car has a tendency to slip up the road then : 3

$$\frac{v^2 \sin 2\theta}{r(1 - \cos 2\theta)} > g$$



(d) The equation of the normal to the hyperbola $xy = c^2$ at $P\left(cp, \frac{c}{p}\right)$ is given by $p^3x - yp = cp^4 - c$.

(i) The normal at P meets the hyperbola again at $Q\left(cq, \frac{c}{q}\right)$.

Show that $p^3q + 1 = 0$. 3

(ii) Show that $PQ = c\left(p^2 + \frac{1}{p^2}\right)\sqrt{p^2 + \frac{1}{p^2}}$. 2

End of Question 13

Question 14 (15marks)

Use a NEW sheet of paper

(a) Using the substitution $x = \tan\theta$, find $\displaystyle\int \frac{d\theta}{\cos\theta\sin\theta + 1}$. 3

(b) A particle is moving on a horizontal straight line. The distance x metres from the origin after t seconds is given by the formula $x = 3\sin 2t + 3\sqrt{3}\cos 2t$.

(i) Show that the motion is simple harmonic. 1

(ii) Find an expression for v^2 in terms of x, where v represents the velocity of the particle. 2

(c) (i) Express the derivative of $\cos^{n-1} x \sin x$ in terms of $\cos x$. 1

(ii) Hence, or otherwise, show that : 2

$$\int_0^{\frac{\pi}{2}} \cos^n x \; dx = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \cos^{n-2} x \; dx.$$

(iii) Evaluate : 2

$$\int_0^{\frac{\pi}{2}} \cos^6 x \; dx$$

(d) A bag contains 3 black discs and 4 white discs. Two players, X and Y, play a game where they swap turns and randomly select a single disc from the bag. The first player to draw a white disc wins. Player X is the first person to select a disc from the bag.
Find the probability of player X winning if :

(i) the disc is returned to the bag after each selection. 2

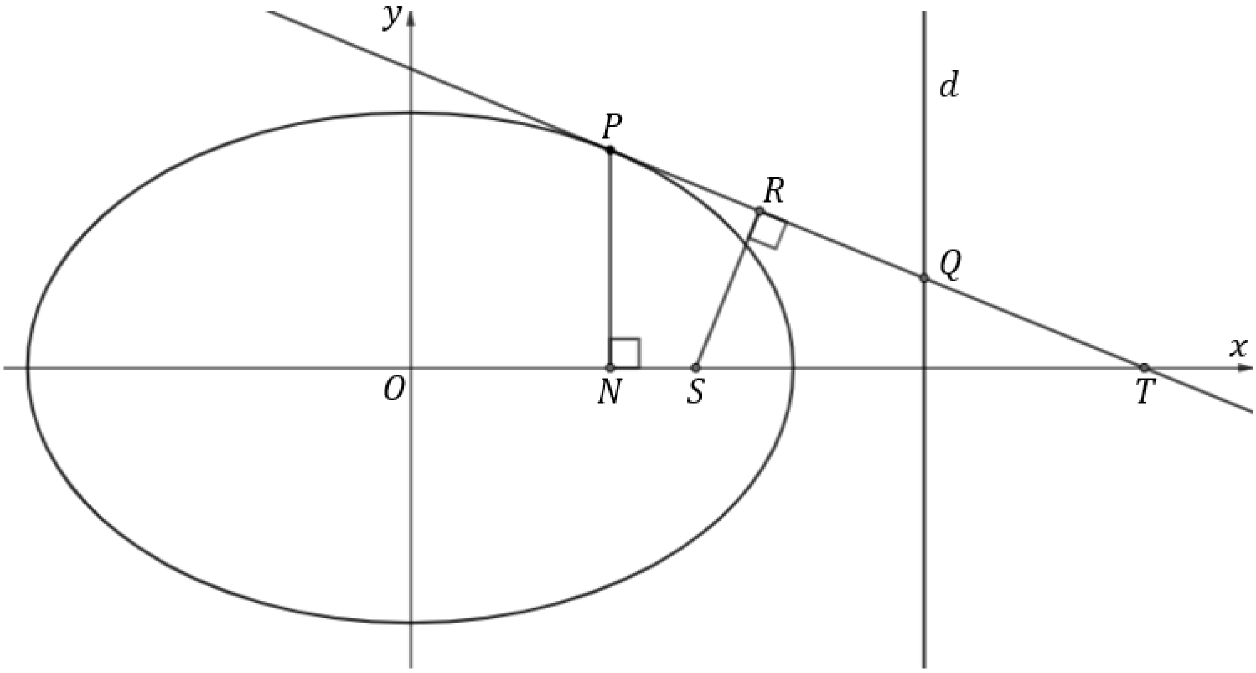
(ii) the disc is NOT returned to the bag after each selection. 2

End of Question 14

Question 15 (15 marks)

Use a NEW sheet of paper

(a) The tangent at the point $P(x_1, y_1)$ on the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the major axis at T and the directrix d at Q. PN is perpendicular to the x −axis and S is the focus. SR is perpendicular to PQ.



Show that :

(i) the equation of the tangent is $\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$. 1

(ii) $a^2 = ON \times OT$. 1

(iii) $\angle PSQ = 90°$. 2

(iv) the lines through OP and RS intersect on the directrix d. 2

Question 15 continues on the next page

Question 15 (continued)

(b) (i) Prove that $2\cos\theta\,(\cos\theta + i\sin\theta) = 1 + \cos 2\theta + i\sin 2\theta$. 1

(ii) Prove that if n is an integer then :

$$-2^n\cos^n\left(\frac{\pi}{n}\right) = \left(1 + \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)^n.$$ 2

(iii) Simplify :

$$\left(1 + \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)^n - \left(1 + \cos\frac{2\pi}{n} - i\sin\frac{2\pi}{n}\right)^n.$$ 1

(c) (i) Show that $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$. 2

(ii) Solve $\cos x - \sin x = \sin 3x$ for $0 \le x \le 2\pi$. 3

End of Question 15

Question 16 (15marks)

Use a NEW sheet paper

(a) (i) Show that $\cot\theta + \frac{1}{2}\tan\frac{\theta}{2} = \frac{1}{2}\cot\frac{\theta}{2}$. 2

(ii) Prove by mathematical induction that for integers $n \geq 1$: 3

$$\sum_{r=1}^{n} \frac{1}{2^{r-1}} \tan\frac{x}{2^r} = \frac{1}{2^{n-1}} \cot\frac{x}{2^n} - 2\cot x \, .$$

(b) z is a complex number such that $z^3 = i(z-1)^3$. 3

Show that $z = \frac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta - 1}$ where $\theta = \frac{4k\pi+\pi}{6}$ for $k = 0,1,2$.

(c) Let $I_n = \int_0^1 \frac{dx}{\left(1+x^2\right)^n}$ for $n > 1$.

(i) Prove that $\int \frac{x}{\left(1+x^2\right)^n} dx = \frac{1}{2(1-n)\left(1+x^2\right)^{n-1}} + c$ for $n > 1$. 2

(ii) Simplify $\frac{1}{(1+x^2)^{n-1}} - \frac{x^2}{(1+x^2)^n}$. 1

(iii) Show that $I_n = \frac{2n-3}{2(n-1)} I_{n-1} + \frac{1}{2^n(n-1)}$. 2

(iv) Show that $I_n \geq 2^{-n}$ for $n > 1$. 2

End of paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics Extension 2

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2+4=?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct → B

A ✗ B ✗ C ○ D ○

Student Number: EXT 2.

Completely fill the response oval representing the most correct answer.

1. A ● B ○ C ○ D ○
2. A ○ B ● C ○ D ○
3. A ○ B ○ C ○ D ●
4. A ● B ○ C ○ D ○
5. A ○ B ● C ○ D ○
6. A ● B ○ C ○ D ○
7. A ● B ○ C ○ D ○
8. A ● B ○ C ○ D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ● C ○ D ●

TRIAL HSC 2019: Mathematics Extension 2 - Solutions

1. $$\int \frac{dx}{\sqrt{-x^2-2x}} = \int \frac{dx}{\sqrt{1-(x^2+2x+1)}}$$

$$= \int \frac{dx}{\sqrt{1-(x+1)^2}}$$

$$= \sin^{-1}(x+1) + C$$

(A)

2. $xy = \frac{8}{9}$ $\quad x+y = \pm\sqrt{2} \times \frac{\sqrt{8}}{3}$

$x+y = \pm\frac{4}{3}$

(B)

3. $x = \alpha, \beta, \gamma$ $\quad y = \frac{1}{\sqrt{\alpha}}, \frac{1}{\sqrt{\beta}}, \frac{1}{\sqrt{\gamma}}$ $\quad y = \frac{1}{\sqrt{x}}$ $\quad x = \frac{1}{y^2}$

$$\left(\frac{1}{y^2}\right)^3 - 3\left(\frac{1}{y^2}\right)^2 + 4\left(\frac{1}{y^2}\right) - 1 = 0$$

$$1 - 3y^2 + 4y^4 - y^6 = 0$$

i.e. $y^6 - 4y^4 + 3y^2 - 1 = 0$

(D)

4. $y = \sqrt{36-9x^2}$ where $36 - 9x^2 \neq 0$ i.e. excluding $x = \pm 2$.

Semi-ellipse with intercepts $y = 6$ and $(x = \pm 2)$

↑ excluded points

(A)

5. $$i^{4x-1} = i^{4x} \times i^{-1}$$

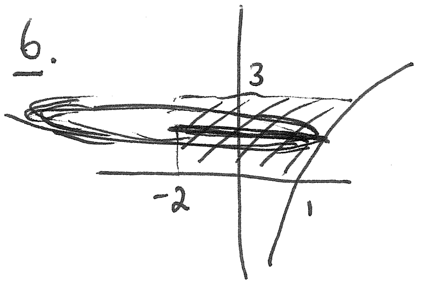
$$= (i^4)^x \times \frac{1}{i} \times \frac{i}{i}$$

$$= 1 \times \frac{i}{-1}$$

$$= -i$$

(B)

6.



$\delta V = \pi(x+2)^2 \delta y$

$V = \pi \int (x+2)^2 dy$

$= \pi \int_0^3 (e^y + 2)^2 dy$

$y = \log_e x$

$x = e^y$

(A)

7. $(1-i) = \sqrt{2}\,\text{cis}(-\frac{\pi}{4})$

(A)

8. $f(x) = x^{n+1}(1 + x + x^2 + \dots + x^{n-1})$

Factored polynomial with (n+1)th root at $x = 0$. This root has horizontal inflexion point if n+1 is odd. Hence, n must be even.

(A)

9. $(1-i)^2 = 1 - 2i + i^2$

$= -2i$

$\therefore 1 - i = \sqrt{-2i}$ $\quad \therefore P(\sqrt{-2i}) = 0$

(D)

10. Let $f(x) = ax^3 + bx^2 + cx + d$.

$f'(x) = 3ax^2 + 2bx + c$

To ensure only one root, we need $f'(x) \geq 0$ for all x or $f'(x) \leq 0$ for all x.

i.e. positive / negative definite

$\Delta = 4b^2 - 4(3a \times c)$

$= 4(b^2 - 3ac)$

$\Delta \leq 0$

$4(b^2 - 3ac) \leq 0$

$b^2 - 3ac \leq 0$

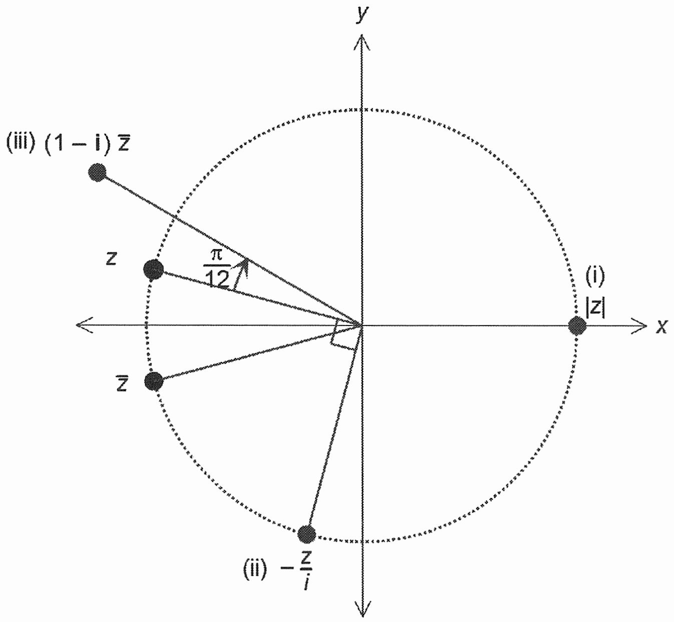
$b^2 \leq 3ac$

(D)

Note: (B) was also accepted though (D) is preferred.

Question 11

	Solution	Comments
(a)(i)	$w - z = 2 + 3i - (3 - 2i)$ $= 2 + 3i - 3 + 2i$ $= -1 + 5i$	Generally well done.
(a)(ii)	$\dfrac{\lvert z\rvert}{w} = \dfrac{\sqrt{(3)^2 + (-2)^2}}{2 + 3i}$ $= \dfrac{\sqrt{13}}{2 + 3i} \times \dfrac{2 - 3i}{2 - 3i}$ $= \dfrac{(2\sqrt{13} - 3\sqrt{13}i)}{4 + 9}$ $= \dfrac{2\sqrt{13}}{13} - \dfrac{3\sqrt{13}}{13}i$	A number of students mistakenly used $\lvert z\rvert = 3 - 2i$. Some students made errors calculating $\lvert z\rvert$ either due to leaving off the root sign in the formula or incorrectly calculating $\sqrt{(3)^2 + (-2)^2} = 5$.
(b)	$x^3 + 4x^2 - 3x - 18 = 0$ Let $P(x) = x^3 + 4x^2 - 3x - 18$ and Let the roots be $\alpha,\ \alpha$ and β. For the double root, $P(\alpha) = P'(\alpha) = 0$. Since $P'(x) = 3x^2 + 8x - 3$, $3\alpha^2 + 8\alpha - 3 = 0$ $(3\alpha - 1)(\alpha + 3) = 0$ $\alpha = \frac{1}{3}$ or $\alpha = -3$ Since the double root must be a factor of 18, $\alpha = -3$ is the double root. By sum of roots : $\alpha + \alpha + \beta = -4$ $\beta = -4 - 2(-3)$ $\beta = 2$ Hence, the roots are $x = -3, -3, 2$.	Generally well done though some students did not end up with a double root after making errors. This should have been a clear indicator of mistakes within your working.
(c)	$x^3 = (ax + b)(x + 1)(x - 2) + c(x - 2) + d(x + 1)$ Let $x = -1$ $\quad -1 = -3c \quad c = \frac{1}{3}$ Let $x = 2$ $\quad 8 = 3d \quad d = \frac{8}{3}$ Consider coefficient of x^3. $\quad a = 1$ Let $x = 0$ $\quad 0 = -2b - 2c + d$ $2b = -2\left(\frac{1}{3}\right) + \frac{8}{3}$ $b = 1$ $I = \int \left(x + 1 + \frac{1}{3}\left(\frac{1}{x + 1}\right) + \frac{8}{3}\left(\frac{1}{x - 2}\right)\right) dx$ $\therefore I = \frac{x^2}{2} + x + \frac{1}{3}\ln(x + 1) + \frac{8}{3}\ln(x - 2) + C$	Some students chose to carry out polynomial division and then break apart the remainder function. This approach was applied in a clumsy manner by many students using this method. There were a range of calculating errors whilst finding the values of the constants a, b, c, d.

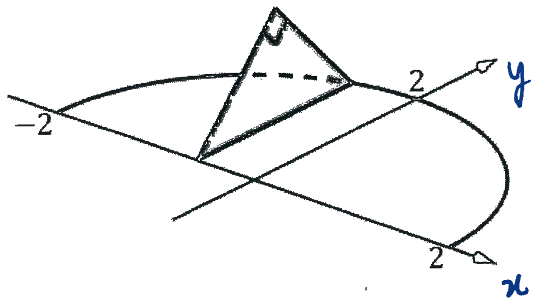
(d)	(ii) $-\frac{z}{i}\times\frac{i}{i}=\frac{-iz}{-1}=iz$ Hence, rotate z by 90° in an anti-clockwise direction. (iii) $1-i=\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ Hence, rotate the vector $\bar{z}$ clockwise by $\frac{\pi}{4}$ and expand the length by $\sqrt{2}$.  	(i) Many students failed to interpret the question correctly – you were required to plot the point representing $\|z\|$. Many students just labelled the length of the vector to A. The value of $\|z\|$ is a real positive number which must be placed on the real axis. (ii) Many students located the correct positioning though it would have been ideal if a circle was shown to reflect that this point and point A were equidistant from the origin. Additionally, the solution should have shown a 90° rotation in the anti-clockwise direction from A. (iii) Poorly executed by many. The argument of z given in the question should have been considered as should have the position of $\bar{z}$. Students should have recognised that the point to be plotted was $\sqrt{2}$ times further away from the origin than A and the point should have been rotated 15° in a clockwise direction from A (or 45° from $\bar{z}$).
(e)	$AD^2 = BD \times CD$ (tangent squared equals prod. of int.) $AD^2 = (BC + CD) \times CD$ $AD^2 = BC \times CD + CD^2$ $\therefore AD^2 - CD^2 = BC \times CD$	Whilst most students were able to work through this question, some students were unable to identify the steps needed to create this proof and made limited progress. Students should ensure they justify with reasoning where appropriate e.g. (tangent squared equals prod. of int.).

Question 12 (15marks)

Use a new sheet of paper

(a) The base of a solid is the region enclosed by the semi-circle $y = \sqrt{4 - x^2}$ and the x axis. Each cross-section perpendicular to the x axis is a right angled isosceles triangle as shown in the diagram.

Find the volume of this solid. 3



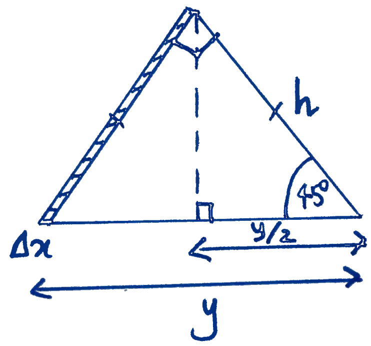
Cross-sectional area:

$A(x) = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times \frac{y}{\sqrt{2}} \times \frac{y}{\sqrt{2}}$

$= \frac{1}{4} y^2$

$= \frac{1}{4}(4 - x^2)$ ✓



$\cos 45° = \frac{y/2}{h}$

$\frac{h}{\sqrt{2}} = \frac{y}{2}$

$h = \frac{y\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}}$

$h = \frac{2y}{2\sqrt{2}}$

$h = \frac{y}{\sqrt{2}}$

(*): $V = \lim_{\Delta x \to 0} \sum_{x=-2}^{2} \frac{1}{4}(4 - x^2) \cdot \Delta x$

$= 2\int_0^2 \frac{1}{4}(4 - x^2)\,dx$ ✓

$= \frac{1}{2}\left[4x - \frac{x^3}{3}\right]_0^2$

$= \frac{1}{2}\left[8 - \frac{8}{3}\right]$

$= \frac{8}{3}$ units^3 ✓

Most error occurred in calculating the cross-sectional area

* X2 students should include this line when finding volume.

(b) Find the gradient of the tangent to the curve $x^2 + xy^2 = 12$ at the point $(3, -1)$. 2

$$x^2 + xy^2 = 12$$

$$2x + x \cdot 2y \cdot \frac{dy}{dx} + 1 \cdot y^2 = 0 \quad \text{(by implicit differentiation)}$$

$$2x + 2xy \cdot \frac{dy}{dx} + y^2 = 0 \quad ✓$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y^2)}{2xy}$$

At $(3,-1)$:

$$m_T = -\frac{(2(3)+(-1)^2)}{2(3)(-1)}$$

$$m_T = \frac{-7}{-6} = \frac{7}{6} \quad ✓$$

(c) Find : 3

$$\int \frac{x}{x^2 - 4x + 6}\, dx$$

$$I = \frac{1}{2} \int \frac{2x-4+4}{x^2-4x+6}\, dx$$

$$= \frac{1}{2} \int \frac{2x-4}{x^2-4x+6}\, dx + \frac{4}{2} \int \frac{1}{(x-2)^2+2}\, dx \quad ✓$$

$$= \frac{1}{2} \ln\left|x^2-4x+6\right| ^{*} + \sqrt{2} \tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right) + C$$

✓ ✓

* Students should note that |absolute value| is used when integration involves logs

Question 12 (continued)

(d) The roots of $5x^3 + 7x^2 - 3x + 2 = 0$ are α, β and γ. 2

Find the value of :

$$(\alpha+\beta)^2 + (\alpha+\gamma)^2 + (\beta+\gamma)^2$$

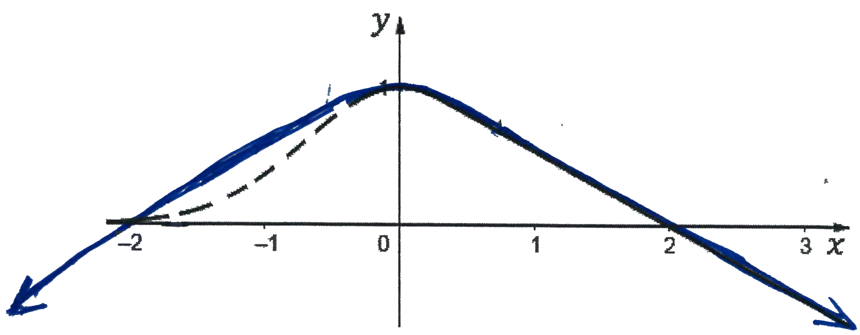
$$(\alpha+\beta)^2+(\alpha+\gamma)^2+(\beta+\gamma)^2$$
$$=\alpha^2+2\alpha\beta+\beta^2+\alpha^2+2\alpha\gamma+\gamma^2+\beta^2+2\beta\gamma+\gamma^2$$
$$=2(\alpha^2+\beta^2+\gamma^2)+2(\alpha\beta+\beta\gamma+\alpha\gamma)$$
$$=2\left[(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)\right]+2(\alpha\beta+\beta\gamma+\alpha\gamma)$$
$$=2(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$$
$$=2\left(-\frac{7}{5}\right)^2-2\left(-\frac{3}{5}\right)$$
$$=\frac{98}{25}+\frac{6}{5}$$
$$=\frac{128}{25}.$$

$$\alpha+\beta+\gamma=-\frac{b}{a}=-\frac{7}{5}$$
$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}=-\frac{3}{5}$$
$$\alpha\beta\gamma=-\frac{d}{a}=-\frac{2}{5}.$$

(e) The diagram below shows the graph of the function $y = f(x)$. The x –axis is an asymptote for $x < 0$.

Draw a separate half page graph for each of the following functions, showing all asymptotes and intercepts.

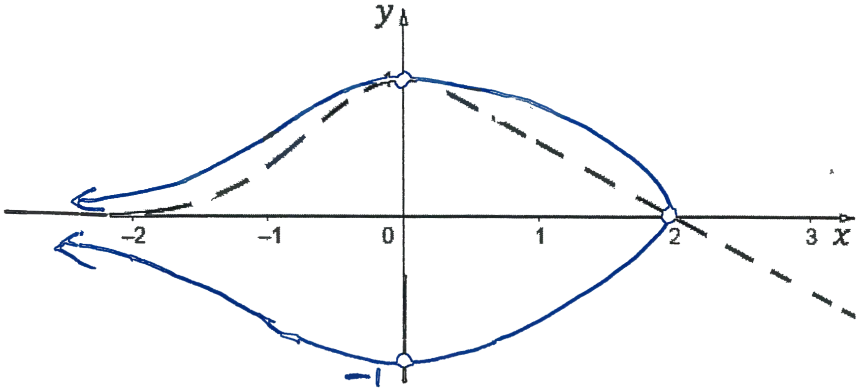
(i) $y = f(|x|)$ 1



reflect $y=f(|x|)$ for $x \geq 0$ in the y-axis.

(ii) $y^2 = f(x)$ 2

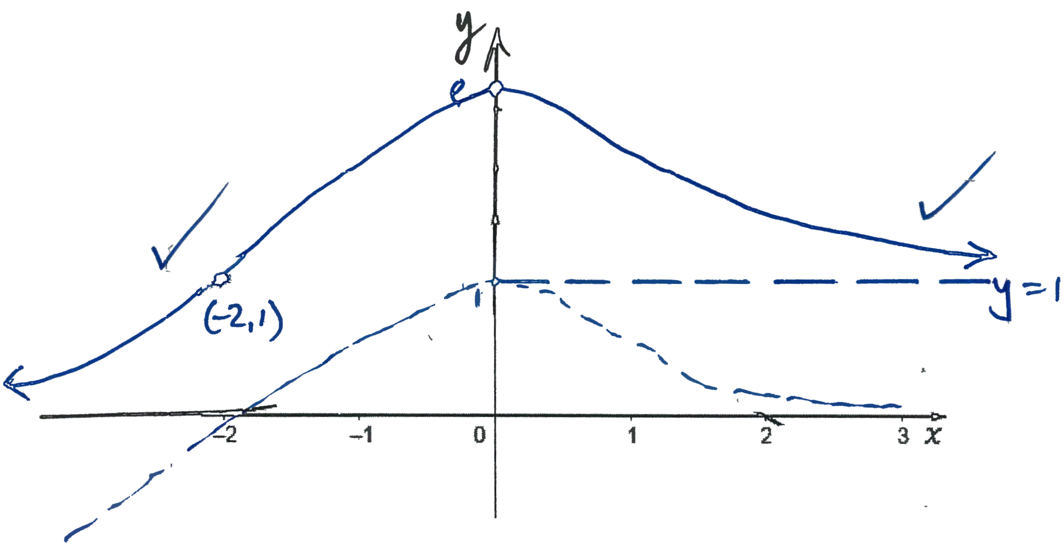
$y \geq 0$: $y = \sqrt{f(x)}$ + reflection in x-axis.



As $x \to -\infty$, $y = \pm\sqrt{f(x)} \to 0$. ✓

intercepts/general shape ✓

(iii) $y = e^{f(-x)}$ 2



- $f(x) \to f(-x)$: flip in y-axis.
- as $f(-x) \to 0$, $e^{f(-x)} \to 1$ (horizontal asymptote)
- as $f(-x) \to -\infty$, $e^{f(-x)} \to 0$.
- when $f(-x) = 1$, $e^{f(-x)} = e^1 = e$.

13 (a) $V_{shell} = \pi(R^2 - r^2)h$

$= \pi((7-y)^2 - (7-y-\delta y)^2) \times 2x$

$= 2\pi \times 2(7-y)\,\delta y\, x$

$= 4\pi(7-y)x\,\delta y$

$V_{solid} = 4\pi \int_0^6 (7-y)\dfrac{\sqrt{36-y^2}}{2}\,dy$

$= 2\pi \int_0^6 (7-y)\sqrt{36-y^2}\,dy$

$= 14\pi \int_0^6 \sqrt{36-y^2} - 2\pi \int_0^6 y(36-y^2)^{\frac{1}{2}}\,dy$ → This is a quadrant not a semi circle.

$= 14\pi \times \frac{1}{4} \times \pi \times 6^2 + \frac{2\pi}{3}\left[(36-y^2)^{\frac{3}{2}}\right]_0^6$

$= 126\pi^2 - \frac{2\pi}{3} \times 216$

$= 126\pi^2 - 144\pi$

(b)(i) $x^2 - 2\sqrt{6}x + 6 + y^2 = 8$

$x^2 + y^2 = 2 + 2\sqrt{6}x$

$\sqrt{6} - 2\sqrt{2} \le x \le \sqrt{6} + 2\sqrt{2}$

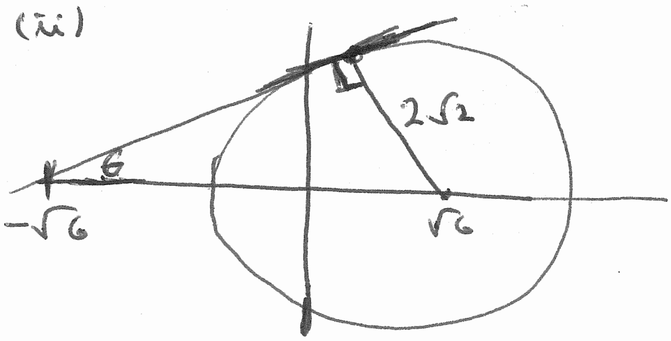
min $|z|$ when $x = \sqrt{6} - 2\sqrt{2}$ and $y = 0$

$\therefore |z| = |(\sqrt{6} - 2\sqrt{2})^2|$

$= 2\sqrt{2} - \sqrt{6}$

$\sqrt{6} - 2\sqrt{2} < 0$ cannot be correct.

(ii)



$\sin\theta = \dfrac{2\sqrt{2}}{2\sqrt{6}}$

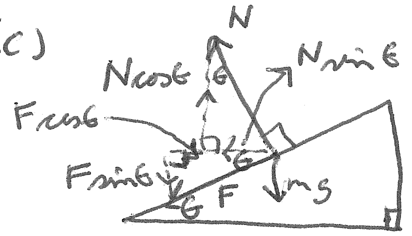
$\theta = 0.61547970\ldots^c$

(iii) $|z - (\sqrt{6} + 2\sqrt{2})|^2 + |z - (\sqrt{6} - 2\sqrt{2}|^2 = (2 \times 2\sqrt{2})^2$

$= 32$

Expanding brackets resulted in little success.

(c)



$$N\cos\theta - F\sin\theta = mg$$

$$N\sin\theta + F\cos\theta = \frac{mv^2}{r}$$

$$F = \frac{mv^2\cos\theta}{r} - mg\sin\theta$$

$$F > 0$$ ⟶ Many forgot to specify this

$$\therefore \frac{mv^2\cos\theta}{r} > mg\sin\theta$$

$$\frac{v^2\cos\theta}{r\sin\theta} > g$$

$$\frac{v^2\cos\theta\sin\theta}{r\sin^2\theta} > g$$

$$\frac{v^2 \times \frac{1}{2}\sin 2\theta}{r\left(\frac{1-\cos 2\theta}{2}\right)} > g$$

$$\therefore \frac{v^2\sin 2\theta}{r(1-\cos 2\theta)} > g$$

(d) (i) $cp^3q - c\frac{p}{q} = cp^4 - c$

$$\cancel{c}p^3q^2 - \cancel{c}p = \cancel{c}p^4q - \cancel{c}q$$

$$p^3q^2 - p^4q = p - q$$

$$p^3q(q-p) = (p-q)$$

$$p^3q = -1$$

(ii) $PQ^2 = (cp - cq)^2 + \left(\frac{c}{p} - \frac{c}{q}\right)^2$

$$= c^2\left((p-q)^2 + \left(\frac{q-p}{pq}\right)^2\right)$$

$$= c^2\left(p^2 - 2pq + q^2 + \frac{p^2 - 2pq + q^2}{p^2q^2}\right)$$

$$= c^2\left(p^2 - 2p \times \frac{-1}{p^3} + \left(\frac{-1}{p^3}\right)^2 + \frac{p^2 + \frac{2}{p^2} + \frac{1}{p^6}}{p^2 \times \frac{1}{p^6}}\right)$$

$$= c^2\left(p^2 + \frac{2}{p^2} + \frac{1}{p^6} + p^6 + 2p^2 + \frac{1}{p^2}\right)$$

$$PG = \sqrt{c^2\left(p^2 + \frac{1}{p^2}\right)^3}$$

$$= c\left(p^2 + \frac{1}{p^2}\right)\sqrt{p^2 + \frac{1}{p^2}}$$

14 (a) $\frac{dx}{d\theta} = \sec^2\theta$ — The t method does not work here.

$$\frac{dx}{\sec^2\theta} = d\theta$$

$$\int \frac{dx}{(\cos\theta\sin\theta + 1)\sec^2\theta}$$

$$= \int \frac{dx}{\tan\theta + \sec^2\theta}$$

$$= \int \frac{dx}{x + 1 + x^2}$$

$$= \int \frac{dx}{(x+\frac{1}{2})^2 + \frac{3}{4}}$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2}{\sqrt{3}}(x+\tfrac{1}{2})\right)$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\frac{2\tan\theta + 1}{\sqrt{3}} + c$$

(b) (i) $\dot{x} = 6\cos 2t - 6\sqrt{3}\sin 2t$

$$\ddot{x} = -12\sin 2t - 12\sqrt{3}\cos 2t$$

$$= -4x$$

(ii) $\frac{v^2}{2} = -4\int x\,dx$

$$v^2 = -4x^2 + c$$

$t = 0,\ x = 3\sqrt{3},\ v = 6$ — Many forgot to find c.

$$36 = -4\times(3\sqrt{3})^2 + c$$
$$= -108 + c$$

$\therefore \quad v^2 = 144 - 4x^2$

(c) (i) $\cos^{n-1}x \cdot \cos x + \sin x \cdot (n-1)\cos^{n-2}x \cdot -\sin x$

$= \cos^n x - \sin^2 x\,(n-1)\cos^{n-2}x$

$= \cos^n x - (1-\cos^2 x)(n-1)\cos^{n-2}x$

$= \cos^n x - (n-1)\cos^{n-2}x + (n-1)\cos^n x$

$= n\cos^n x - (n-1)\cos^{n-2}x$

(ii) $\int_0^{\frac{\pi}{2}} n\cos^n x = \left[\cos^{n-1}x\sin x\right]_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2}x\,dx$

$= 0 + (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2}x\,dx$

$\int_0^{\frac{\pi}{2}} \cos^n x = \frac{n-1}{n}\int_0^{\frac{\pi}{2}} \cos^{n-2}x\,dx.$

(iii) $I_6 = \frac{5}{6} I_4$

$= \frac{5}{6} \times \frac{3}{4} I_2$

$= \frac{5}{8} \times \frac{1}{2} \int_0^{\frac{\pi}{2}} dx.$

$= \frac{5}{8} \times \frac{1}{2} \left[x\right]_0^{\frac{\pi}{2}}$

$= \frac{5\pi}{32}$

(d) (i) $\frac{4}{7} + \frac{3}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{4}{7} + \ldots\ldots$

$= \frac{4}{7}\left(1 + \left(\frac{3}{7}\right)^2 + \left(\frac{3}{7}\right)^4 + \left(\frac{3}{7}\right)^6 + \ldots\right)$

$= \frac{4}{7} \times \frac{1}{1-\left(\frac{3}{7}\right)^2}$

$= \frac{7}{10}$

Many confused replacement with non-replacement.

(ii) $\frac{4}{7} + \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5}$

$= \frac{28}{35}$

Question 15

(a)(i) $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{2x}{a^2} \times \frac{b^2}{2y}$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$m_{Tangent} = -\frac{b^2x_1}{a^2y_1} \quad \text{at } P(x_1, y_1)$$

Equation of the tangent

$$y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$$

$$a^2y_1y - a^2y_1^2 = -b^2x_1x + b^2x_1^2$$

$$b^2x_1x + a^2y_1y = b^2x_1^2 + a^2y_1^2$$

$$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$ since $P(x_1, y_1)$ is on the ellipse.

(ii) $\frac{x_1x}{a^2} = 1$ where $y = 0$.

$$x = \frac{a^2}{x_1}$$

$$O(0,0) \;\; N(x_1, 0) \; T\left(\frac{a^2}{x_1}, 0\right)$$

$$ON \times OT = x_1\left(\frac{a^2}{x_1}\right)$$

$$ON \times OT = a^2$$

(iii) $\frac{x_1}{ae} + \frac{y_1 y}{b^2} = 1$ where $x = \frac{a}{e}$

$$\frac{y_1 y}{a^2(1-e^2)} = \frac{ae - x_1}{ae}$$

$$y = \frac{(ae - x_1)(a - ae^2)}{ey_1}$$

$$P(x_1, y_1)\ S(ae, 0)\ Q\left(\frac{a}{e}, \frac{(ae - x_1)(a - ae^2)}{ey_1}\right)$$

$$m_{SP} = \frac{y_1}{x_1 - ae}$$

$$m_{SQ} = \frac{(ae - x_1)(a - ae^2)}{ey_1} \div \left(\frac{a}{e} - ae\right)$$

$$m_{SQ} = \frac{(ae - x_1)(a - ae^2)}{ey_1} \times \frac{e}{a - ae^2}$$

$$m_{SQ} = \frac{ae - x_1}{y_1}$$

$$m_{SP} \times m_{SQ} = \frac{y_1}{x_1 - ae} \times \frac{ae - x_1}{y_1}$$

$$m_{SP} \times m_{SQ} = \frac{-(x_1 - ae)}{x_1 - ae}$$

$$m_{SP} \times m_{SQ} = -1$$

Therefore $\angle PSQ = 90°$

(iv) $m_{PO} = \frac{y_1}{x_1}$ $\quad O(0,0)$

$m_{RS} = \frac{a^2 y_1}{b^2 x_1}$ $\quad S(ae, 0)$

Equation PO: $\quad y = \frac{y_1}{x_1} x$

Equation RS: $\quad y = \frac{a^2 y_1}{b^2 x_1}(x - ae)$

$$\frac{y_1}{x_1} x = \frac{a^2 y_1}{b^2 x_1}(x - ae)$$

$$x\left(1 - \frac{a^2}{b^2}\right) = -\frac{a^3 e}{b^2}$$

$$x = -\frac{a^3 e}{b^2} \times \frac{b^2}{b^2 - a^2}$$

$$x = -\frac{a^3 e}{a^2(1 - e^2) - a^2}$$

$$x = -\frac{a^3 e}{a^2 - a^2 e^2 - a^2}$$

$$x = \frac{a^3 e}{a^2 e^2}$$

$$x = \frac{a}{e}$$

Which is on the directrix d.

(b)(i) $LHS = 2\cos^2\theta + 2i\sin\theta\cos\theta$

$$= 2\left(\frac{1}{2} + \frac{\cos 2\theta}{2}\right) + i\sin 2\theta$$

$$= 1 + \cos 2\theta + i\sin 2\theta$$

$$= RHS$$

(ii) $RHS = \left(2\cos\left(\frac{\pi}{n}\right)\left(\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}\right)\right)^n$

$$= 2^n\cos^{\mathrm{n}}\left(\frac{\pi}{n}\right)(\cos\pi + i\sin\pi)$$

$$= 2^n\cos^n\left(\frac{\pi}{n}\right)(-1)$$

$$= -2^n\cos^n\left(\frac{\pi}{n}\right)$$

$$= LHS$$

(iii) $\left(1 + \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)^n - \left(1 + \cos\frac{2\pi}{n} - i\sin\frac{2\pi}{n}\right)^n$

$$= \left(1 + \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}\right)^n - \left(1 + \cos\left(-\frac{2\pi}{n}\right) + i\sin\left(-\frac{2\pi}{n}\right)\right)^n$$

$$= -2^n\cos^n\left(\frac{\pi}{n}\right) - -2^n\cos^n\left(-\frac{\pi}{n}\right)$$

Students need to show that with a negative argument then result from the previous part can be applied.

$$= -2^n\cos^n\left(\frac{\pi}{n}\right) + 2^n\cos^n\left(\frac{\pi}{n}\right)$$

$$= 0$$

(c)(i) $\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$

$\sin(\theta-\phi) = \sin\theta\cos\phi - \sin\phi\cos\theta$

$\sin(\theta+\phi) + \sin(\theta-\phi) = 2\sin\theta\cos\phi$

Let $A = \theta + \phi$

and $B = \theta - \phi$

$A + B = 2\theta \qquad A - B = 2\phi$

$\theta = \dfrac{A+B}{2} \qquad \phi = \dfrac{A-B}{2}$

Therefore $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

(ii) $\cos x = \sin 3x + \sin x$

$\cos x = 2\sin 2x\cos x$

$\cos x\,(2\sin 2x - 1) = 0$

$\cos x = 0 \qquad \sin 2x = \dfrac{1}{2}$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2} \qquad 2x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}$

Many students did not use the result from the first part to simplify the equation. Some students did not get all the solutions.

Question 16

(a) (i) $LHS = \cot\theta + \frac{1}{2}\tan\frac{\theta}{2}$

$$= \frac{1}{\tan\theta} + \frac{1}{2}\tan\frac{\theta}{2}$$

$$= \frac{1-\tan^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}} + \frac{1}{2}\tan\frac{\theta}{2}$$

$$= \frac{1}{2\tan\frac{\theta}{2}} - \frac{\tan\frac{\theta}{2}}{2} + \frac{1}{2}\tan\frac{\theta}{2}$$

$$= \frac{1}{2}\cot\frac{\theta}{2}$$

$\therefore LHS = RHS$

(i) The easier approach to this question was to make use of t formulae.
Some students made it more complicated and time-consuming than necessary.

(ii) Generally well done but many students made errors in proving the result was true for $n = 1$, particularly in recognising that $LHS = \tan\frac{x}{2}$ and not $\frac{1}{2}\tan\frac{x}{2}$.
Some students got confused whilst attempting to prove the result was true for $n = k + 1$.
In many cases, solutions provided could have been presented more clearly. Key steps in this section of the proof were required. Explicit reference to the result from part (i) was preferable.

Question 16

(a) (ii) ① Prove true for n=1.

$$LHS = \sum_{r=1}^{1} \frac{1}{2^{r-1}} \tan\frac{x}{2^r} = \frac{1}{2^0}\tan\frac{x}{2^1} = \tan\frac{x}{2}$$

$$RHS = \frac{1}{2^0}\cot\frac{x}{2^1} - 2\cot x = \cot\frac{x}{2} - 2\cot x = 2\left(\cot x + \frac{1}{2}\tan\frac{x}{2}\right) - 2\cot x$$

using part (i)

$$= \tan\frac{x}{2}$$

LHS = RHS ∴ proven true for n=1.

② Assume true for n=k.

i.e. assume $\sum_{r=1}^{k} \frac{1}{2^{r-1}}\tan\frac{x}{2^r} = \frac{1}{2^{k-1}}\cot\frac{x}{2^k} - 2\cot x$

③ Prove true for n=k+1.

i.e. prove $\sum_{r=1}^{k+1} \frac{1}{2^{r-1}}\tan\frac{x}{2^r} = \frac{1}{2^k}\cot\frac{x}{2^{k+1}} - 2\cot x$.

$$LHS = \frac{1}{2^{k-1}}\cot\frac{x}{2^k} - 2\cot x + \frac{1}{2^k}\tan\frac{x}{2^{k+1}}$$

$$= \frac{1}{2^{k-1}}\left(\cot\frac{x}{2^k} + \frac{1}{2}\tan\frac{x}{2\times 2^k}\right) - 2\cot x$$

$$= \frac{1}{2^{k-1}}\left(\frac{1}{2}\cot\frac{x}{2\times 2^k}\right) - 2\cot x \quad \text{using part (i)}$$

$$= \frac{1}{2^k}\cot\frac{x}{2^{k+1}} - 2\cot x$$

LHS = RHS If true for n=k, proven true for n=k+1.

Since proven true for n=1, then proven true for integers $n \geq 1$ by mathematical induction.

Question 16

(b) $z^3 = i(z-1)^3$

$$\left(\frac{z}{z-1}\right)^3 = i$$

Let $\frac{z}{z-1} = r\,\text{cis}\,\theta$.

$$(r\,\text{cis}\,\theta)^3 = \text{cis}\,\frac{\pi}{2}$$

$$r^3\,\text{cis}\,3\theta = \text{cis}\,\frac{\pi}{2}$$

$r^3 = 1 \quad 3\theta = \frac{\pi}{2} + 2k\pi$ where k is an integer

$r = 1 \quad \theta = \frac{\pi + 4k\pi}{6}$ where $k = 0, 1, 2$

$\frac{z}{z-1} = \text{cis}\,\theta$

$$z = z\,\text{cis}\,\theta - \text{cis}\,\theta$$

$$z(\text{cis}\,\theta - 1) = \text{cis}\,\theta$$

$$z = \frac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta - 1}$$

where $\theta = \frac{4k\pi + \pi}{6}$ for $k = 0, 1, 2$.

Using the method above, some students did not show clearly how $z = \frac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta - 1}$ developed in their solution. For full marks, the bottom part of the solution above was needed.

An alternative pathway taken successfully by a minority of students was to substitute

$$z = \frac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta - 1}$$

into the equation $z^3 = i(z-1)^3$ and then solve for θ. This was effectively done by most students who used this method.

A common approach was to substitute $z = \cos\theta + i\sin\theta$ into the original form of the equation. However, unsuccessful attempts did not get further than :

$$(\cos\theta + i\sin\theta)^3 = i(\cos\theta + i\sin\theta - 1)^3$$

Question 16

(c) $I_n = \int_0^1 \frac{dx}{(1+x^2)^n} \quad n > 1$

(i) Let $u = 1+x^2 \quad du = 2x\,dx$

$$I = \frac{1}{2}\int \frac{du}{u^n}$$

$$= \frac{1}{2}\int u^{-n}\,du$$

$$= \frac{1}{2} \times \frac{u^{-n+1}}{-n+1} + c$$

$$= \frac{1}{2(1-n)\,u^{n-1}} + c$$

$$= \frac{1}{2(1-n)(1+x^2)^{n-1}} + c$$

This was one of the easier parts of Q16 but some students did not recognise the substitution required. Also, this was a "show" question and required appropriate working to justify the result of the integration.

(ii) $$\frac{1}{(1+x^2)^{n-1}} - \frac{x^2}{(1+x^2)^n} = \frac{1+x^2 - x^2}{(1+x^2)^n}$$

$$= \frac{1}{(1+x^2)^n}$$

This should have been an easy mark to obtain. Whilst generally well done, unfortunately some students did not attempt the question.

Question 16

(c) (iii)

$$I_n = \int_0^1 \frac{1}{(1+x^2)^{n-1}}\,dx - \int_0^1 \frac{x^2}{(1+x^2)^n}\,dx$$

By parts: $u = x$, $v' = \frac{x}{(1+x^2)^n}$, $u' = 1$, $v = \frac{1}{2(1-n)(1+x^2)^{n-1}}$

$$= I_{n-1} - \left(\left[\frac{x}{2(1-n)(1+x^2)^{n-1}}\right]_0^1 - \int_0^1 \frac{1}{2(1-n)(1+x^2)^{n-1}}\,dx\right)$$

$$= I_{n-1} - \frac{1}{2(1-n)2^{n-1}} + \frac{1}{2(1-n)} I_{n-1}$$

$$= \frac{2(1-n)+1}{2(1-n)} I_{n-1} + \frac{1}{2(n-1)2^{n-1}}$$

$$\therefore I_n = \frac{3-2n}{2-2n} I_{n-1} + \frac{1}{2^n(n-1)}$$

$$I_n = \frac{2n-3}{2(n-1)} I_{n-1} + \frac{1}{2^n(n-1)}$$

Many students did not make the link to the result from part (ii) – though this was not necessary to carry out the proof. There were many attempts with a varied degree of success.

A common approach taken was to use integration by parts in the original form of the integral, letting $u = \frac{1}{(1+x^2)^n}$ and $v' = 1$. This resulted in the following :

$$I_n = \frac{1}{2^n} + 2nI_n - 2nI_{n+1}$$

To prove the result required, students needed to make use of the substitution $N = n + 1$.

Question 16

(c) (iv) For $0 \le x \le 1$ $\quad 0 \le x^2 \le 1$

$1 \le 1 + x^2 \le 2$

Since $1+x^2 \le 2$, $\quad \frac{1}{1+x^2} \ge \frac{1}{2}$

For $n > 1$ $\quad \left(\frac{1}{1+x^2}\right)^n \ge (2^{-1})^n$

$$\therefore \int_0^1 \frac{dx}{(1+x^2)^n} \ge \int_0^1 2^{-n}\, dx$$

$$I_n \ge 2^{-n}\left[x\right]_0^1$$

Hence $I_n \ge 2^{-n}$

Most students did not make any significant progress on this question.

In addition to the solution above, the other approaches that attracted marks included :

- use of mathematical induction
- an argument relating to area under the curve $y = \frac{1}{(1+x^2)^n}$ and the area of a rectangle inside that region